

Theorem Fig

$$az + bz^2 + cz^3 + dz^4 + ez^5 + fz^6 + gz^7 -$$

$$= a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1} + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b^3 z^{m+3} + \frac{m}{1} a^{m-1} c + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} c^2 + \frac{m}{1} a^{m-1} c^2$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-5} b^5 z^{m+5} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{1} a^{m-4} b^3 c + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 d + \frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{2} a^{m-3} b c^2 + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} b e + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} c d + \frac{m}{1} a^{m-1} f$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \times \frac{m-5}{6} a^{m-6} b^5 c + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-5} b^4 c^2 + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-4} b^3 c^3 + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-3} b^2 c^4 + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-2} b c^5 + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-1} c^6 + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-1} c^7$$

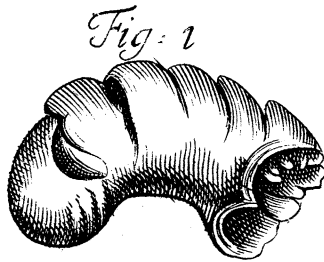
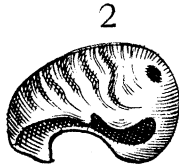
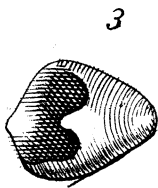
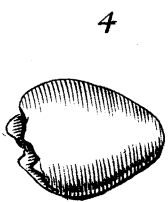


Fig. 5. Transact. N^o 230.

$$\overline{gz^7 + hz^8 + iz^9 \& C} \Big| \overset{m}{=}$$

$$a^{m-3}b^3z^{m+3} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} a^{m-4}b^4z^{m+4}$$

$$x^{m-2}bc + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3}b^2c$$

$$x^{m-1}d + \frac{m}{1} \times \frac{m-1}{1} a^{m-2}bd$$

$$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2}c^2$$

$$+ \frac{m}{1} a^{m-1}e$$

$$\frac{m-4}{5} \times \frac{m-5}{6} a^{m-6}b^6z^{m+6} \& C$$

$$\frac{m-3}{4} \times \frac{m-4}{1} a^{m-5}b^4c$$

$$\frac{m-2}{3} \times \frac{m-3}{1} a^{m-4}b^3d$$

$$\frac{m-2}{1} \times \frac{m-3}{2} a^{m-4}b^2c^2$$

$$\frac{m-1}{2} \times \frac{m-2}{1} a^{m-3}b^2e$$

$$\frac{m-1}{1} \times \frac{m-2}{1} a^{m-3}bcd f$$

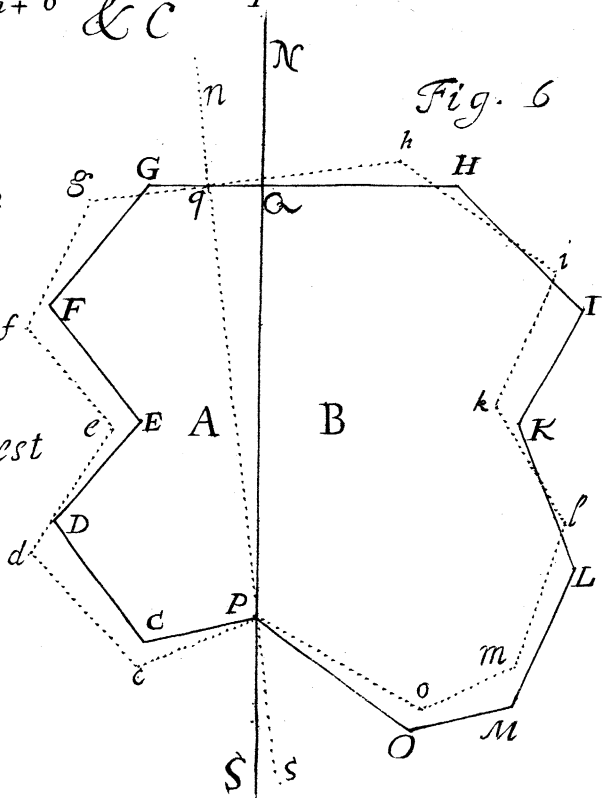
$$\frac{m}{1} \times \frac{m-1}{1} a^{m-2}bf$$

$$\frac{m-1}{2} \times \frac{m-2}{3} a^{m-3}c^3 \text{ West}$$

$$\frac{m}{1} \times \frac{m-1}{1} a^{m-2}ce$$

$$\frac{m}{1} \times \frac{m-1}{2} a^{m-2}d^2$$

$$+ \frac{m}{1} a^{m-1}g$$



that whatever is said of one may be said of t'other. However I think to give sometime a more formal Demonstration of it.

See the Theorem *Fig. 5.*

IV. *A Demonstration of an Error committed by common Surveyors in comparing of Surveys taken at long Intervals of Time arising from the Variation of the Magnetick Needle, by William Molyneux Esq; F. R. S.*

THE Variation of the Magnetick Needle is so commonly known, that I need not insist much on the Explication thereof, 'tis certain that the true Solar Meridian, and the Meridian shewn by a Needle, agree but in very few places of the World; and this too, but for a little time (if a Moment) together. The Difference between the true Meridian and Magnetick Meridian perpetually varying and changing in all Places and at all Times; sometimes to the Eastward and sometimes to the Westward.

On which account 'tis impossible to compare two Surveys of the same place, taken at distant times, by Magnetick Instruments (such as the *Circumferentor*, by which the *Down Survey*, or Sir *William Petty's Survey of Ireland* was taken) without due allowance be made for this Variation. To which purpose we ought to know the Difference between the Magnetick Meridian and true Meridian at that time of the *Down Survey*, and the said Difference at the time, when we make a New Survey to compare with the *Down Survey*.

Ent

But here I would not be understood, as if I proposed hereby to shew, that a Map of the same place, taken by Magnetick Instruments at never so distant times, should not at one time give the same *Figure* and *Contents* as at another time. This certainly it will do most exactly, the variation of the Needle having nothing to do either in the *Shape* or *Contents* of the Survey. All that is affected thereby, is, the Bearings of the Lines run by the Chain, and the Boundaries between Neighbours. And how this may cause a considerable Error (unless due allowance be made for it) is what I shall prove most fully.

In order to which, let us suppose that about the Year 1657. (at which time the Down Survey was taken) the Magnetick Meridian and true Meridian did agree at *Dublin*, or pretty nigh all over *Ireland*; that is to say, that there was no Variation. And indeed by Experiment it was at that time found, as I am well assur'd, that at *Dublin* it was hardly half a Degree.

Let us suppose that in the Year 1695. the Variation was 7 Degrees from the North to the Westward : that it was really so, I believe I am pretty well assured, from an Experiment thereof made by my self with all diligence. But this is not material, let us now only suppose it.

Let *A B* represent the Survey of two Town Lands, one in the possession of *A*, and t'other in the possession of *B*, which we will call *A* Town-Land and *B* Town-Land, taken by the Down Survey, *Anno* 1657. when there was no Variation.

Let the Line *NS* running through the Point *P* be the true Meridian, and consequently the Magnetick Meridian also at that time, because of the supposed no Variation, and let this Line *NS* be also the Boundary between the two Town-Lands *A* and *B*.

In the Year 1695. when the Variation is 7 Degrees from the North to the Westward, *B* having a Map of the Down Survey, and being suspicious that his Neighbour *A* had incroached on him by a Ditch *PQ*, imployes a Surveyor to inquire into the Matter: The Surveyor finds by his Map that the Boundary between *B* and his Neighbour *A* run from the Point *P* through a Meadow directly according to the Magnetick Meridian *SPN*; but observing the Ditch *PQ* cast up much to the Eastward of the present Magnetick Meridian, he concludes that *A*. has incroached on *B*, and that the Ditch ought to have been cast up alongst the Line *Pq*, the Angle *QPq* being an Angle of 7 Degrees, that is the present Variation of the Needle; and the Line *Pq* the present Magnetick Meridian: For which Variation, not making any allowance, he positively determines that *B*. has all the Land in the Triangle *QPq*, more than he ought to have; and that his Ditch ought to run alongst the Line *Pq*.

'Tis true indeed, if the Surveyor go the whole Surround of the Lands *A* and *B* he will find their Figure and Contents exactly agreeable to the Map here expressed. But then the Bearings of the Lines are all 7 Degrees different from the Bearings in the Map, and they will run in and out upon the adjacent Neighbouring Lands, and cause endless Differences between their Possessors; as is manifest from the Figure: wherein the prickd Lines represent the Disagreement in the Bearings of the Lines, protracted from the Point *P*; and we see *A* incroaching on his Neighbours on the Westward, as he incroaches on *B*, and *B*'s Eastward Neighbours incroaching on him, and so forward and clear round. Whereas, by a due allowance for the Variation of the Needle, all this Confusion and Disagreement is avoided, and every thing hits right.

Thus.

Thus for instance in the Case before us, knowing that the Magnetick Variation has caused the present Magnetick Meridian to fall in the Line $nqPs$, 7 Degrees from the North to the Westward; to Reduce this to the Magnetick Meridian at the time of the Down Survey, I must make the Meridian of my Map to fall 7 Degrees to the Eastward of my Magnetick Meridian; as we see the Line PQ falls 7 Degrees to the Eastward of the Line Pq .

What is here said on supposition that the Magnet had no Variation at the time of the first Survey taken, and that it had 7 Degrees variation Westward at the time of the second Survey, may easily be accommodated to the supposal of any other Variations at the first and second Surveys, *Mutatis mutandis*, for knowing the Variations we know their Difference; and if we know their Difference, this gives us the Angle QPq , by which we reduce them to each other.

The best way therefore to make Maps invariable, constant, and everlasting, were for the Surveyors, who use Magnetick Instruments to make always allowance for the Magnetick Variation, and to protract and lay down their Plats by the true Meridian. This the wary Sailor is fully convinced of: and therefore in Steering his Course, he constantly allows for the present Variation, which he observes by the *Azimuth* Compass, or else he would miss his appointed Harbour oftner then he would hit it: For no two Points on the Globe keep the same Bearing to each other by the Magnetick Meridian for any time together. And though the Variation be flow, yet in a long Course, or in times pretty distant, it may cause vast Errors, unless allowed for. Thus for instance, Suppose in the Year 1660. a Sailor had steered from the Lands-end of *England* to Cape *Fenister* in *Spain*, by his Magnetick Compass a direct South Course; and that

that at that time there were no Variation. Afterwards, *Anno* 1700. when there was (suppose) 7 Degrees of Variation from the North to the Westward, another Sailor intending to make the same Passage, steers directly the same Southerly Course by his Magnetick Compass; I say, this last Seaman will be carried far into the *Bay of Biscay* to the Eastward, and will miss of his desired Port by many Leagues; but if in his Course he hath allowed for this Variation, and instead of sailing a direct Southerly Course by his Compass, he had steer'd 7 Degrees from the South to the Westward, he had hit his Point. Whether these be the true Bearings of these two Places, it matters not: we go on the Supposition that they are.

Perhaps it may be objected, That Surveys may be taken without Magnetick Instruments, and that therefore this Error arising from the Magnetick Variation, and Change of the Bearing of Lines, may be avoided. To which I Answer, first, That granting a Survey may be taken *without* Magnetick Instruments, this is nothing against what we have laid down relating to Surveys that are taken *with* Magnetick Instruments, as the Down Survey actually was, and most Surveys at present actually are taken therewith. Secondly, Though a Survey may be taken truly without Magnetick Instruments, so as to shew the exact Angles and Lines of the Plat, and consequently the true Contents, yet this will not give the true Bearings of the Lines, or shew my Position in relation to my Neighbours, or the other parts of the Country. This must be supply'd by the Magnet, or something equivalent thereto, as finding a true Meridian Line on your Land by Celestial Observation. And I doubt not but the ancient *Ægyptians*, before the Discovery of the Magnet were forced to some such Expedient in their Surveys and Applotments of

Lands, between Neighbour and Neighbour, after the Inundations of the *Nile*, which, we are told, gave the first Original to Geometry and Surveying. Absolute Necessity and Use having introduced these, as Delight and Diversion introduced Astronomy amongst the *Chaldeans*.

And this brings me to another Objection which may be made against the Instance before laid down: It may be said, That certainly the Surveyor which *B* employed was very ignorant, who would choose to judge of the Line *PQ*, rather by its bearing than by determining the Point *Q*, by measuring from *H* and *G*. To this I answer, What if both the Points *H* and *G* were vanish'd since the Down Survey was taken? What if the whole face of the Country were chang'd, save only the Point *P* and the Line *PQ*? How shall the Surveyor then judge of the Line *PQ* but by its bearing? That this is no extravagant Supposition, we have an Example in *Egypt* above-mentioned, where the *Nile* lays all flat before it, and so uniformly covers all with Mud, that there is no distinction. In such a Case your Bearing must certainly help you out, there is no other way.

But I answer secondly, To say that the Surveyor might have determin'd the Point *Q* by admeasurement from *G* and *H*, or any other adjoining noted Points, as from *F, K, I, &c.* 'tis very true; But then 'tis against our Supposition. I am upon shewing an Error that arises from judging of the Line *PQ* by *Magnetick bearing*, and to tell me that this might be avoided by another way, is to say nothing. I my self shew how it may be avoided by allowing for the Variation; but still it is an Error, till it be avoided.

But

But thirdly, If *B*'s Surveyor do not allow for the Variation of the Needle, he will never exactly determine even the Points *G, F, H, K,* &c. or any other Points in the Plat; but instead thereof will fall on the Points *g, h, f, k.*

From what has been laid down, we may see the absolute necessity of allowing for the Variation of the Magnet, in comparing old Surveys with new ones; for want of which great Disputes may arise between neighbouring Proprietors of Lands: and it were to be wish'd that our Honourable and Learned Judges would take this Matter into their Consideration whenever any Business of this kind comes before them. Hitherto an absolute Acquiescence in the Down Survey, without any of the fore-mention'd Allowance, has been agreed upon as a standing Rule in our Courts of Judicature in *Ireland*; but that many Men may be injured thereby, I suppose is manifest from what foregoes.

I have only this to add, That least I be thought herein to strike at the Truth or Exactness of the Down Survey, 'tis not at all the intention of this Paper, but rather to confirm it, by shewing which way Men ought to Examine it truly, and not by the common ways used by them, which rather confound it, and all that claim under it.

See the Table Fig. 6.

Although this Paper was chiefly designed for the ending of Contests in the Kingdom of *Ireland* about the interests of some of those whose Lands are Neighbouring, and have been Surveyed by Magnetick Instruments, yet considering its universal Use, it was thought it would be very grateful to the Curious to publish it here.

V. Extracts

Theorem Fig. 5. Transact. N^o 230.

$$\overbrace{az + bz^2 + cz^3 + dz^4 + ez^5 + fz^6 + gz^7 + hz^8 + iz^9 \& C}^m$$

$$= a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1} + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b^3 z^{m+3} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} a^{m-4} b^4 z^{m+4}$$

$$+ \frac{m}{1} a^{m-1} c + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} bc + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 c + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} b d + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} c^2 + \frac{m}{1} a^{m-1} e$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-5} b^5 z^{m+5}$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{1} a^{m-4} b^3 c$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 d$$

$$+ \frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{2} a^{m-3} b c^2$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} b e$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} c d$$

$$+ \frac{m}{1} a^{m-1} f$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \times \frac{m-5}{6} a^{m-6} b^6 z^{m+6} \& C$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{1} a^{m-5} b^4 c$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{1} a^{m-4} b^3 d$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} \times \frac{m-3}{2} a^{m-4} b^2 c^2$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 e$$

$$+ \frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{1} a^{m-3} b c d$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} b f$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} c^3 \text{ West}$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} c e$$

$$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2} d^2$$

$$+ \frac{m}{1} a^{m-1} g$$

